Mathematics:

Calculations and Manipulatives Policy

Policy Approval Date: September 2019
Policy Review Date: September 2022

Barham Church of England Primary School

Calculation Policy: 2019

Contents

Introduction

The written calculation methods we use in schools have changed a lot over the years.
We have designed this booklet to guide parents, pupils and teachers alike of how written calculation should be taught in the present day, meaning that our children are receiving consistent guidance from all who are involved with their learning.

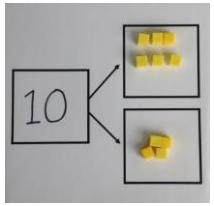
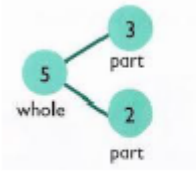
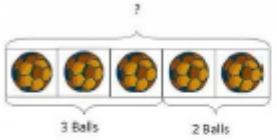
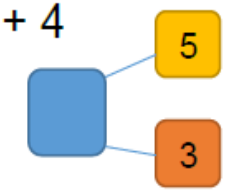
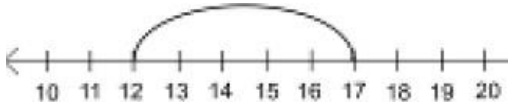
All the methods we use are vital stepping stones for children to develop a secure understanding of how the four mathematical operations work.
We believe that children must have a clear understanding of what the numbers mean when calculating, so we move to more concise approaches when we feel that the child is ready for them.

The year group guidance is just that – a **guide**, and teachers are encouraged to teach children the techniques that they think are appropriate for the child.
This may sometimes mean dipping into the methods advised for slightly younger or older children.

We hope this booklet is of use to you.
If you require any further clarity, please speak with your class teacher or the Maths Subject Leader, Mrs Morhen

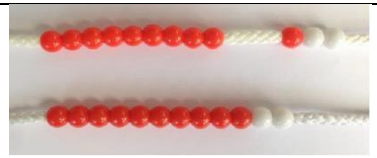
Progression in Calculations

Addition

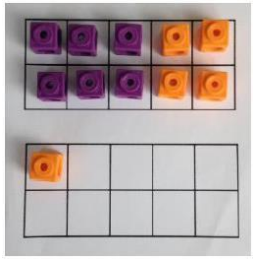
Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model *Pupils will use this method from EYFS onwards*	    Use cubes to add two numbers together as a group or in a bar.	   Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on *Pupils will use this method from Year 1 onwards*	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

Regrouping to make 10.

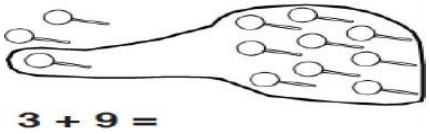
Pupils will use this method from Year 1 onwards



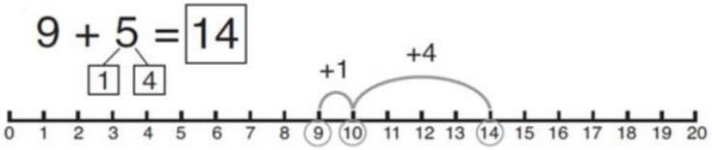
6 + 5 = 11



Start with the bigger number and use the smaller number to make 10.



Use pictures or a number line. Regroup or partition the smaller number to make 10.



7 + 4= 11

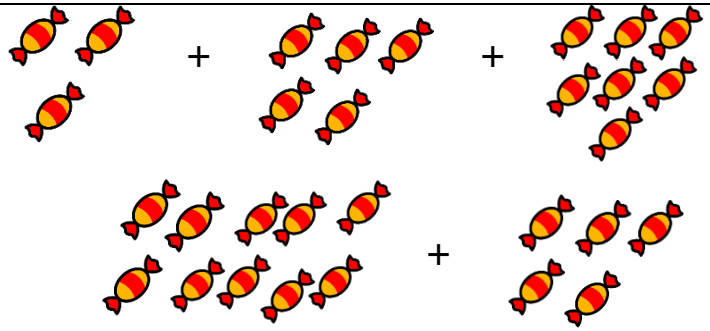
If I am at seven, how many more do I need to make 10. How many more do I add on now?

Adding three single digits

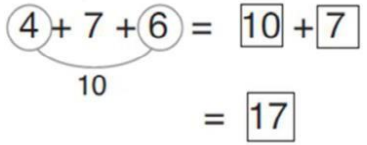
Pupils will use this method from Year 1 onwards

4 + 7 + 6= 17
Put 4 and 6 together to make 10. Add on 7.

Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.



Add together three groups of objects. Draw a picture to recombine the groups to make 10.

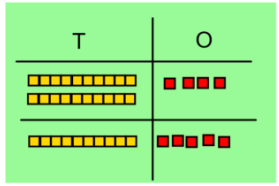


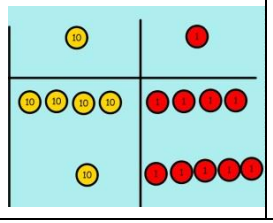
Combine the two numbers that make 10 and then add on the remainder.

Column method- no regrouping

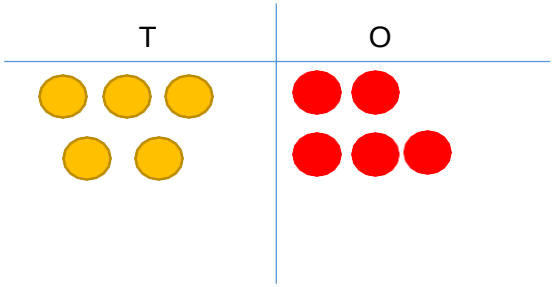
Pupils will use this method from Year 2 onwards

24 + 15=
Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.





After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.

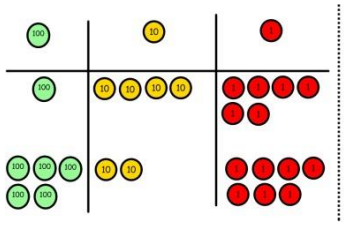
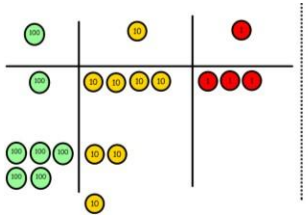
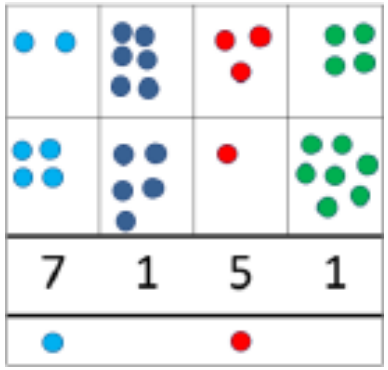


Calculations

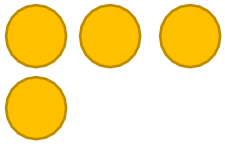
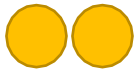
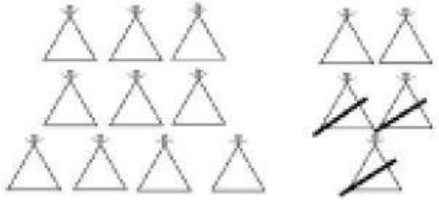
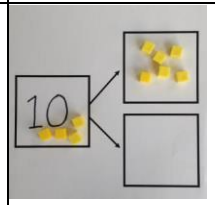
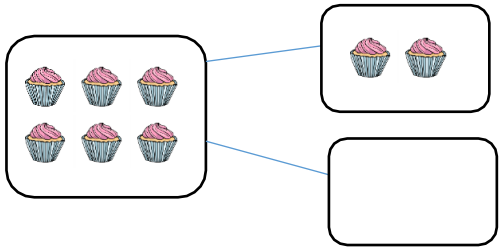
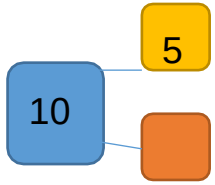
21 + 42 =

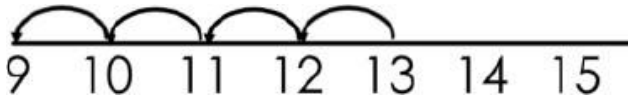
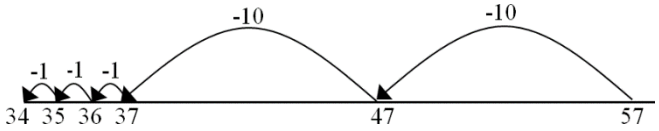
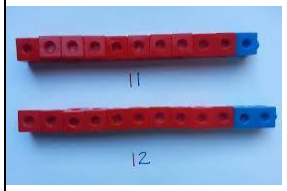
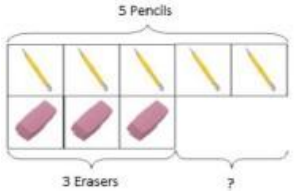
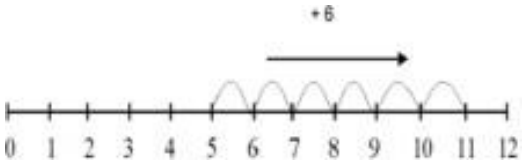
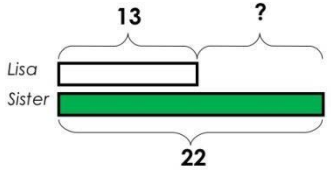
21

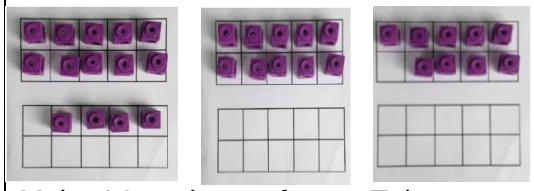
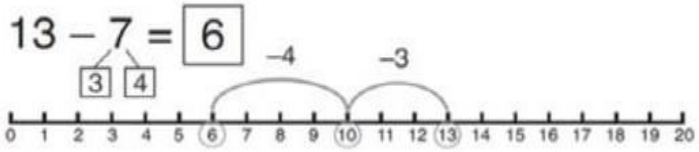
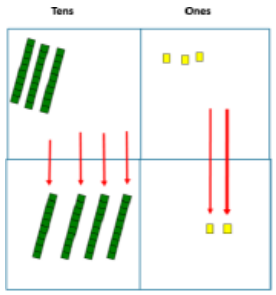
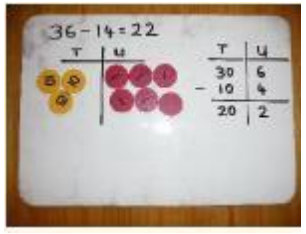
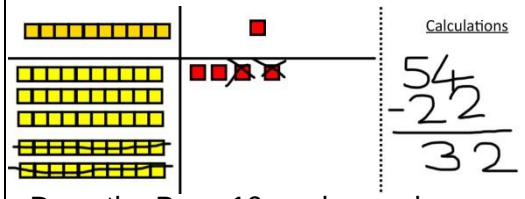
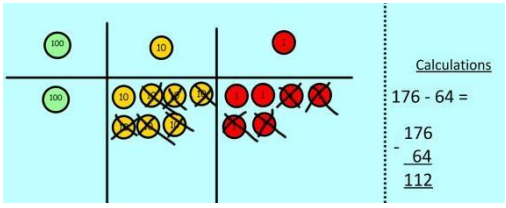
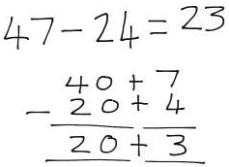
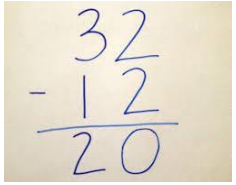
+ 42

Column method- regrouping *Pupils will use this method from Year 2 onwards*	Make both numbers on a place value grid.  146 + 527 Add up the units and exchange 10 ones for one 10.  146 + 527 Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  7 1 5 1	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 536 + 85 621 1 1 72.8 + 54.6 127.4 1 1 £23.59 + £7.55 £31.14 1 1 1 23.361 9.080 59.770 + 1.300 93.511 2 1 2

Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones *Pupils will use this model from EYFS onwards.*	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4  	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6
Part Part Whole Model *Pupils will use this model from EYFS onwards.*	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.

Counting back *Pupils will use this model from Year 1 onwards.*	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.
Find the difference *Pupils will use this model from Year 1 onwards.*	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers. **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.

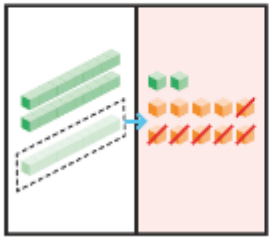
Make 10 *Pupils will use this model from* *Year 1 onwards*	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?
Column method without regrouping *Pupils will use this model from* *Year 2 onwards*	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first.  	 Draw the Base 10 or place value counters alongside the written calculation to help to show working.  	 As the children gain confidence with understanding the manipulatives, they should begin to record in this manner: This will lead to a clear written column subtraction. 

Column method with regrouping

Pupils will use this model from Year 2 onwards

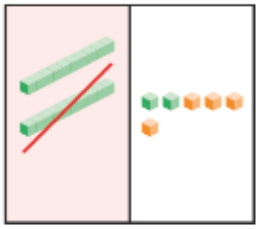
Use Dienes to model regrouping.

Step 1 Regroup 1 ten into 10 ones.
Subtract the ones.
12 ones − 6 ones = 6 ones



	tens	ones
	2 ~~3~~	12 ~~2~~
−	1	6
		6

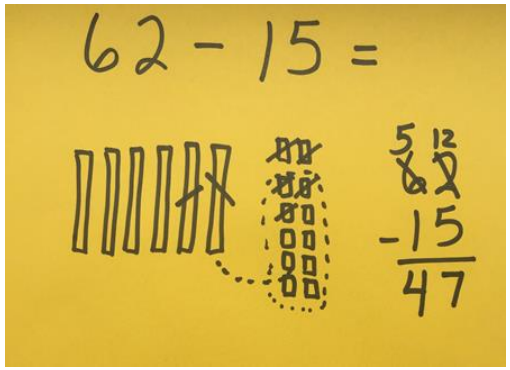
Step 2 Subtract the tens.
2 tens − 1 ten = 1 ten



	tens	ones
	2 ~~3~~	12 ~~2~~
−	1	6
	1	6

32 − 16 = 16

Children can draw a representation of the larger number using place value counters. They can show regrouping by crossing through a ten and drawing an arrow to the 10 ones that have been 'exchanged'.



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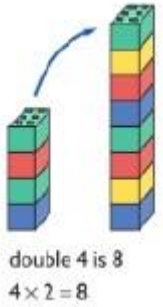
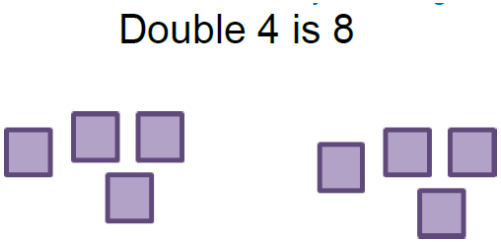
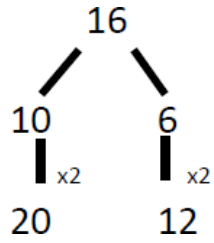
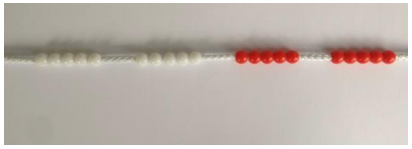
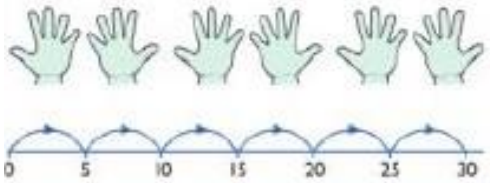
Moving forward the children use a more compact method.

	h	t	o
	~~4~~ 5	1 ~~2~~ 11	~~10~~ 0
−	2	6	9
	2	5	1

This will lead to an understanding of subtracting any number including decimals.

		5 12		1
	2	~~6~~	~~3~~ .	0
-		2	6 .	5
	2	3	6 .	5

Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling *Children will use this model from EYFS onwards*	Use practical activities to show how to double a number.  	Draw pictures to show how to double a number.  	 Partition a number and then double each part before recombining it back together.
Counting in multiples *Children will use this model from Year 1 onwards.*	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

<table>
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Repeated addition

Children will use this model from Year 1 onwards.

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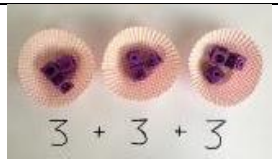

Use different objects to add equal groups.

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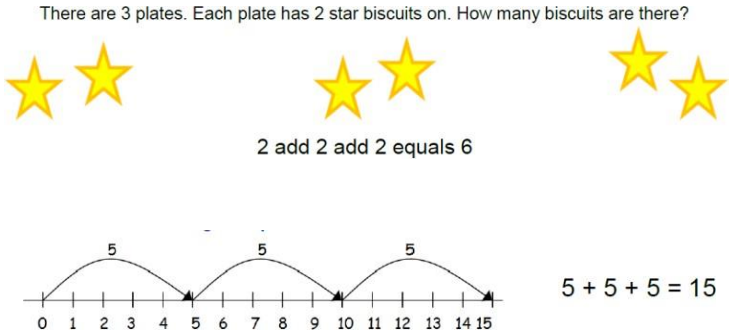

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Write addition sentences to describe objects and pictures.

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Meaning of each factor

(When first developing an understanding of multiplication)

Children will use this model from Year 1 onwards.

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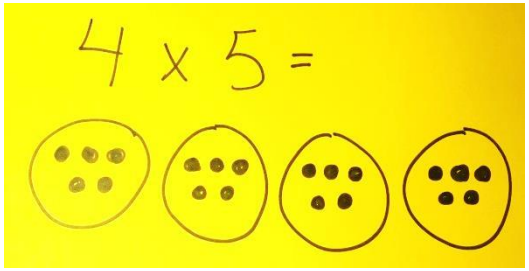
When first introducing multiplication, introduce by explaining that first factor tells us how many groups and the second tells us how many in the group. The product is how many there are altogether.

3 groups of 5 flowers = 15 flowers
3 x 5 = 5 + 5 + 5 = 15

</td>
<td>

Children can draw pictures to represent the meaning of multiplication sentences:



</td>
<td>

After seeing many concrete and pictorial representations, children can move on to saying the meaning of each number in multiplication sentence:

4 x 5 = 20

'There are four groups with 5 in each group which equals 20 altogether'.

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Note:
Once children have developed a basic understanding of multiplication including its commutative nature (see below), it is not necessary to specify the meaning of each factor. As is the practice in Shanghai and Singapore, either factor can be the multiplier or multiplicand eg. 24 x 3 can mean 24 lots of 3 or the number 24 three times. The language of 'multiplied by' needs to be introduced in Year 2 alongside commutivity.

</td>
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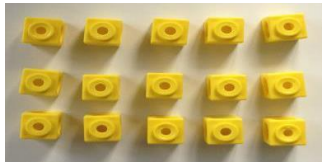
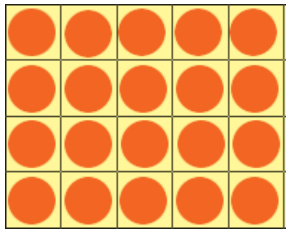
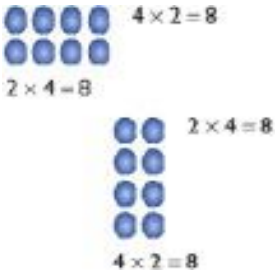
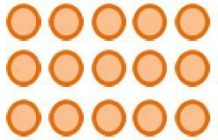
</table>

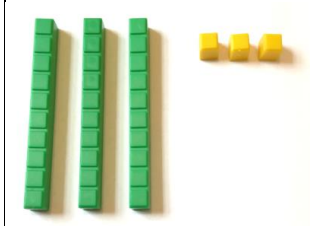
Arrays- showing commutative multiplication *Children will use this model from Year 2 onwards.*	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.   4 × 2 = 8 2 × 4 = 8 2 × 4 = 8 4 × 2 = 8 Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

2 digit Multiplication

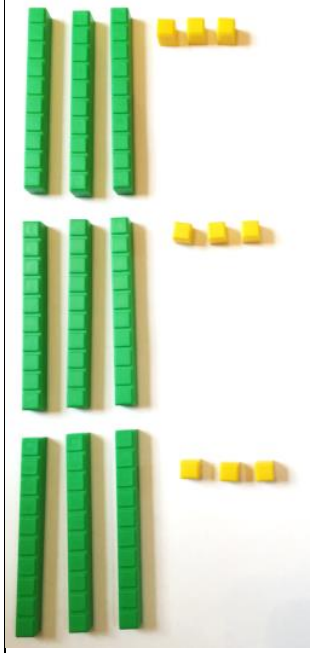
Children will use this model from Year 3 onwards.

When first introducing multiplying 2 digit numbers, Dienes are used to help the children 'see' the whole number that is being multiplied:

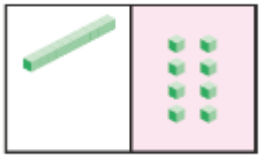
3 x 23 =



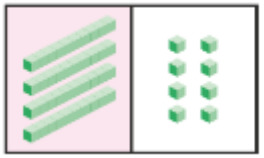
means 3 lots of 23:



We need to multiply 2 ones by 4:

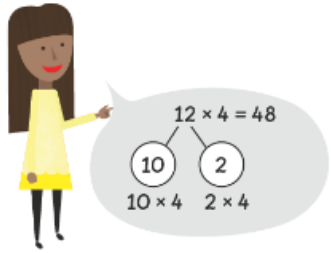


Then, multiply 1 ten by 4:



So, 12 x 4 = 48

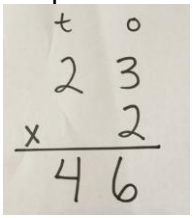
The children can use a part-whole diagram to partition the 2 digit number and multiply each part:



Once this is secure, they record the two calculations in columns:

	t	o
	2	3
×		2
		6
+	4	0
	4	6

And finally, this can be compacted:

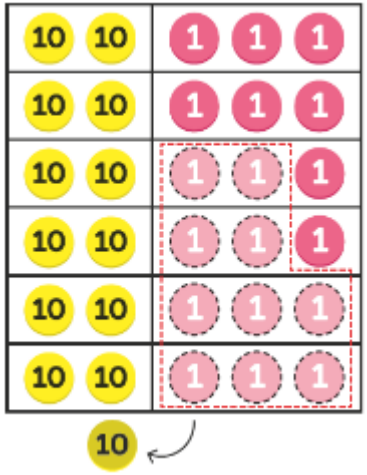


2 and 3 digit multiplication with regrouping

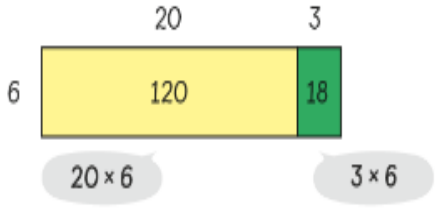
Children will use this model from Year 3 onwards.

Children will use place value counters to represent the numbers:

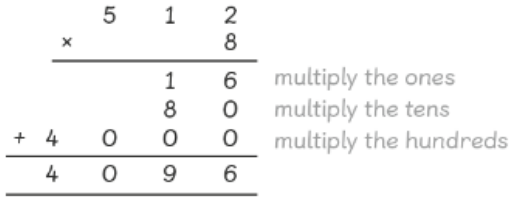
6 x 23 =



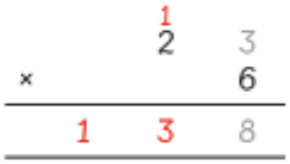
A bar model can also be used to help the children understand what is happening in this calculation:

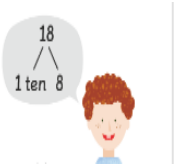


An expanded method should be used to reinforce place value:



Then finally, this can be compacted to:





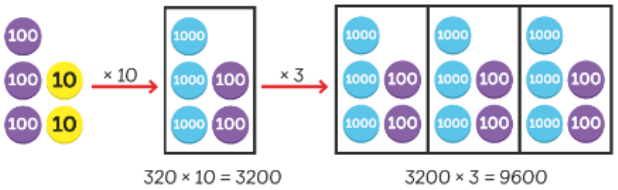
In each Year of KS2 children will should revisit the expanded method in each year group and represent multiplication using the concrete, pictorial abstract approach so that they understand the compact, written method.

Multiplication: 3 digit by 2 digit

Children will use this model from Year 5 onwards.

The children should be encouraged to think about their earlier work partitioning when multiplying. They should also continue to use place value counters to represent the number and to show the place value of the different parts of the calculation:

320 × 31 =



320 × 10 = 3200 3200 × 3 = 9600

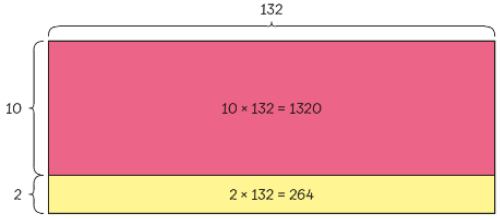
320 × 30 = 9600
320 × 1 = 320
320 × 31 = 9920

Real life situations and the bar model should be used to ensure the children understand the calculation as the compact method is introduced.

1 12 × 132 =

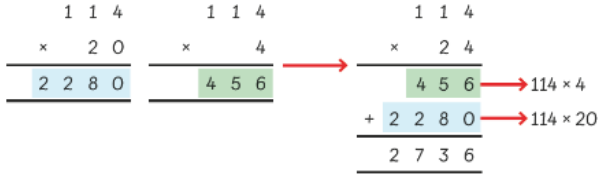
2 times	200	60	4	264
10 times	1000	300	20	1320

12 × 132 = 1320 + 264
= 1584



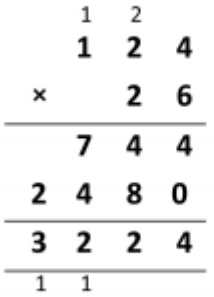
12 × 132 = 1320 + 264
= 1584

The calculation can be broken down into two parts to help make clear to the children what each 'row' means.



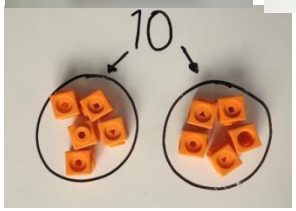
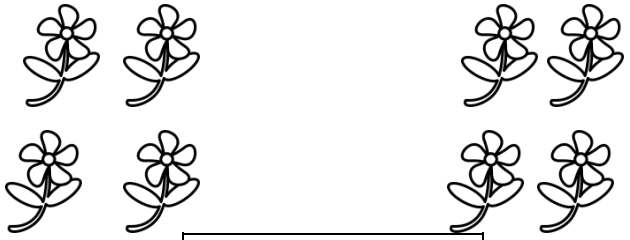
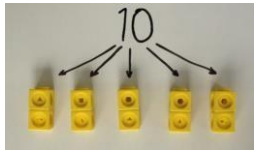
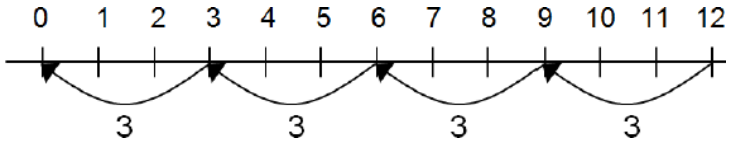
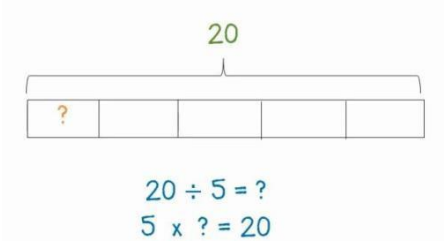
When the children understand the calculation it can be compacted to look like this:

124 × 26 becomes



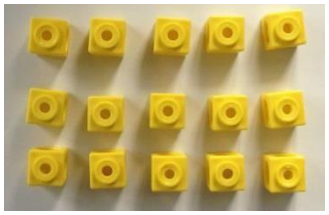
Answer: 3224

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects fairly *Children will do this with equipment in EYFS and Year 1 but will begin to use the notation starting in Year 2.*	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities equally.  8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Division as grouping *Children will do this with equipment in Year 1 but will begin to use the notation starting in Year 2.*	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. 'I have 10 biscuits, I give 2 to each child, how many children can get biscuits?  I have 12 chairs. I put 4 chairs around each table, how many tables do I need? 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

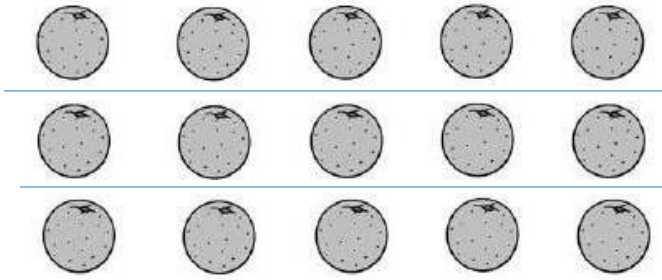
Division within arrays

Children will use this model from Year 2 onwards.



Link division to multiplication by creating an array and thinking about the number sentences that can be created.

Eg 15 ÷ 3 = 5 5 x 3 = 15
15 ÷ 5 = 3 3 x 5 = 15



Draw an array and use lines to split the array into groups to make multiplication and division sentences.

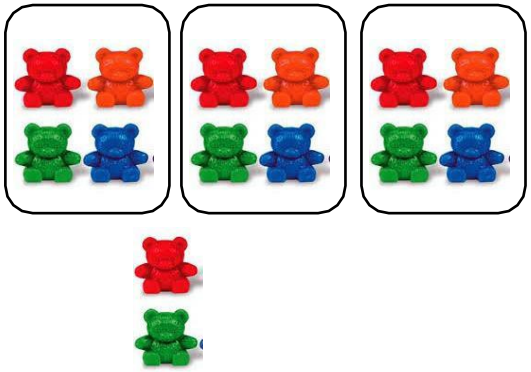
Find the inverse of multiplication and division sentences by creating four linking number sentences.

7 x 4 = 28
4 x 7 = 28
28 ÷ 7 = 4
28 ÷ 4 = 7

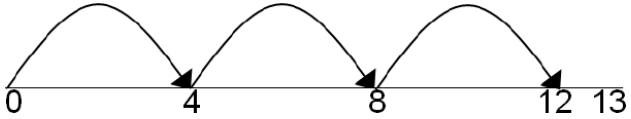
Division with a remainder

(Years 3-6)
** children working at greater depth in Year 2 will cover this.*

14 ÷ 3 =
Divide objects between groups and see how much is left over



Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.

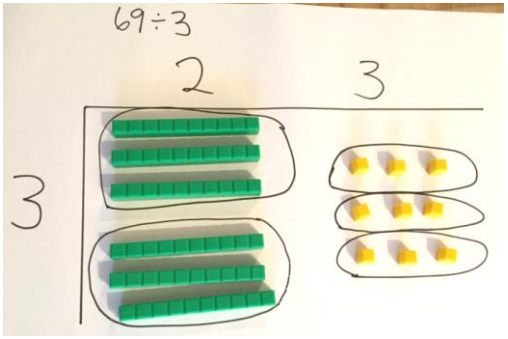
Complete written divisions and show the remainder using r.

29 ÷ 8 = 3 r 5

Short division

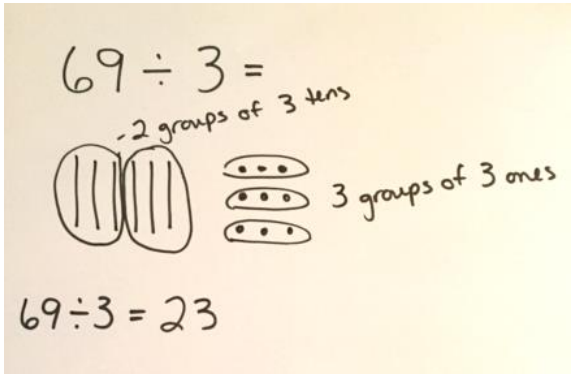
Children will use this model from Year 3 onwards

When dividing 2 digit numbers children begin by representing the number with Dienes. They then see how many groups of the divisor they can make:



I can make 2 groups of 3 tens. I can make 3 groups of 3 ones.

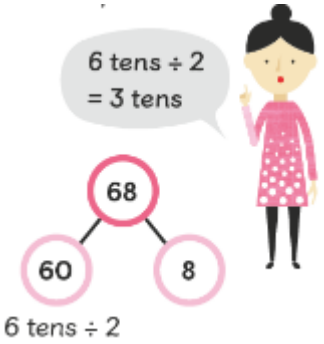
Children can use drawings to represent the Dienes (or they can draw place value counters) and see how they can be grouped:

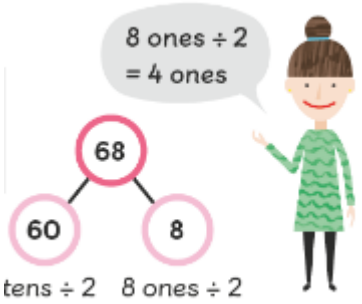


Encourage them to move towards counting in multiples to divide more efficiently.

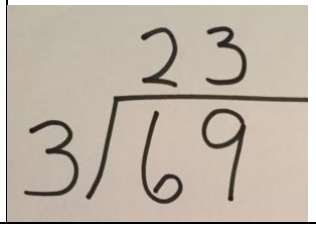
This can link to a mental method of partitioning:

68 ÷2 =





The compact method should continue to be taught alongside using Dienes or place value counters until understanding is secure.



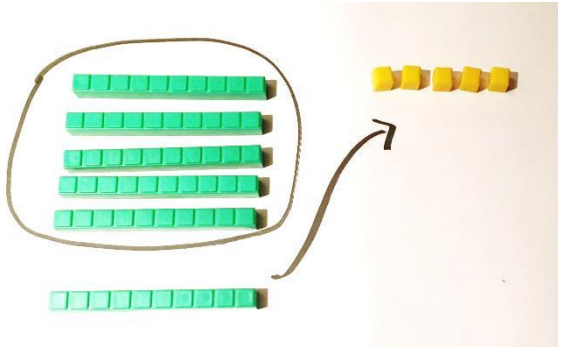
Short division with regrouping

Children will use this model from Year 3 onwards.

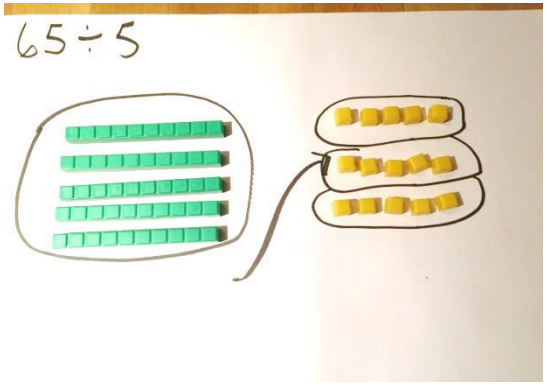
Have children explore calculations like 65 ÷ 5 by making groups of the divisor and discovering that the 'extra' 10 can be regrouped with the ones. How many groups of 5 in 6 (tens)?

65 ÷ 5 =

Step 1:

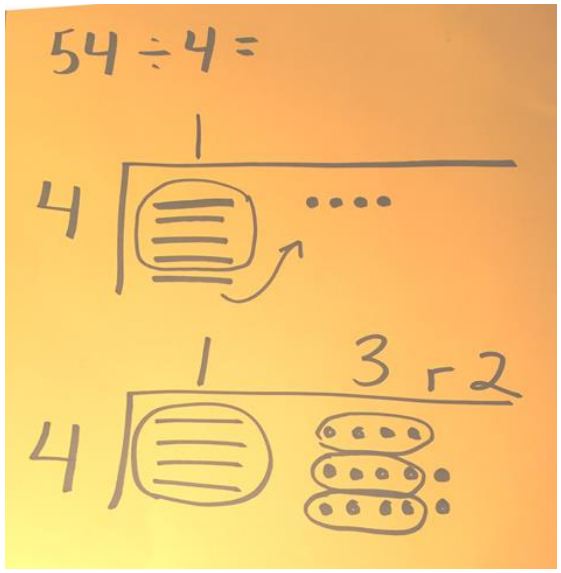


Step 2:



Children can make drawings of the Dienes ('burgers, chips and beans') or place value counters and group them to model short division. The algorithm should be introduced alongside the drawing.

54 ÷ 4 = 13 r 2



Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 045 \\ 8\overline{)\,3^{3}6^{4}0} \end{array}$$

Move onto divisions with a remainder.

362 ÷ 7 =

$$\begin{array}{r} 51\ r5 \\ 7\overline{)\,3\ 6\ {}^{1}2} \end{array}$$

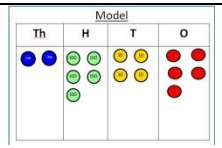
362 ÷ 7 = 51 r5

Finally move into decimal places to divide the total accurately

$$\begin{array}{r} 01.375 \\ 8\overline{)\,1\,{}^{1}1.{}^{3}0{}^{6}0{}^{4}0} \end{array}$$

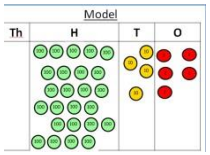
Long division

Children will use this calculation from Year 6 onwards.



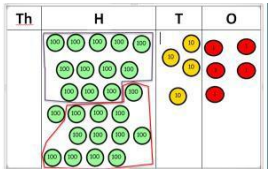
2544 ÷ 12
How many groups of 12 thousands do we have?
None

Exchange 2 thousand for 20 hundreds.



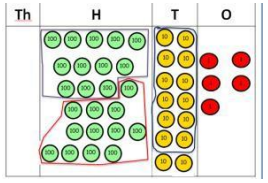
$$12\overline{)2544} \quad 0$$

How many groups of 12 are in 25 hundreds? 2 groups. Circle them.
We have grouped 24 hundreds so can take them off and we are left with one.



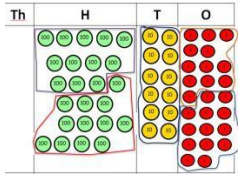
$$12\overline{)2544} \quad 02 \quad 24 \quad 1$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2



$$12\overline{)2544} \quad 021 \quad 24 \quad 14 \quad 12 \quad 2$$

Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2

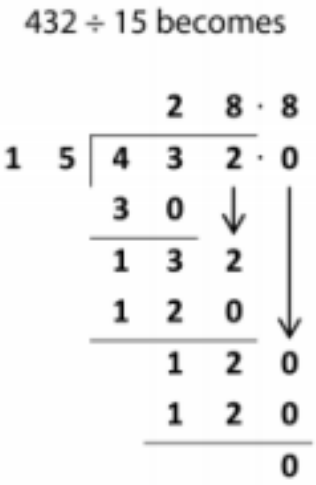


$$12\overline{)2544} \quad 0212 \quad 24 \quad 14 \quad 12 \quad 24 \quad 24 \quad 0$$

Instead of using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books.

Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process.

Children should be encouraged to think of long division as a way of keeping track of the calculations they are already doing mentally when they use the short division method.
Long division should be used when the divisor is a 2 digit number where the mental calculations become too complex to keep track of.



They can write a list of multiples of the divisor to help them or use estimation to see how many groups of 15 there are at each step.