Mathematics:

Calculations and Manipulatives Policy

Policy Approval Date: April 2024
Policy Review Date: April 2027

Barham Church of England Primary School

Calculation Policy: 2024

Contents

Introduction

The written calculation methods we use in schools have changed a lot over the years.
We have designed this booklet to guide parents, pupils and teachers in how written calculation should be taught at Barham CE Primary School, meaning that our children are receiving consistent guidance from all who are involved with their learning.

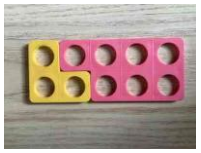
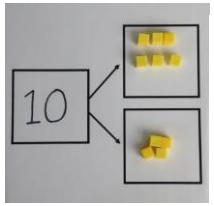
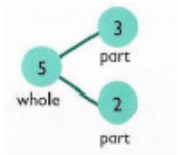
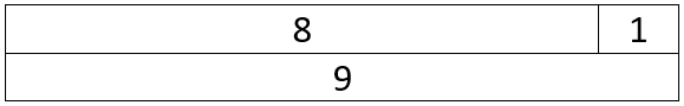
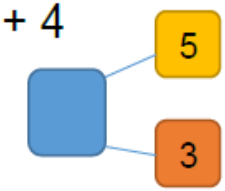
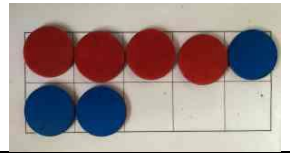
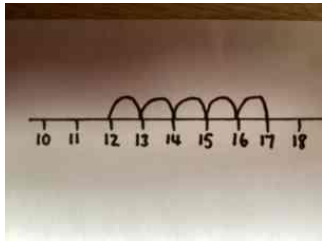
All the methods we use are vital stepping stones for children to develop a secure understanding of how the four mathematical operations work.
We believe that children must have a clear understanding of what the numbers mean when calculating, so we move to more concise approaches when we feel that the child is ready for them.

The year group guidance is just that – a **guide**, and teachers are encouraged to teach children the techniques that they think are appropriate for the child. This may sometimes mean dipping into the methods advised for slightly younger or older children.

If you require any further clarity, please speak with your class teacher or the Maths Subject Leader, Mr Johnston.

Progression in Calculations

Addition

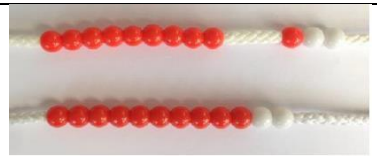
Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model *Pupils will use this method from EYFS onwards*	     Use cubes to add two numbers together as a group or in a bar.	  	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on *Pupils will use this method from Year 1 onwards*	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. Use tens frames or Numicon to visualize and help with counting. 	12 + 5 = 17  Start at the larger number on the number line and count on in ones to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

<table>
<tr>
<td>

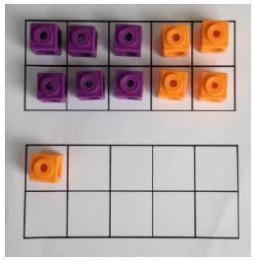
Regrouping to make 10.

Pupils will use this method from Year 1 onwards

</td>
<td>

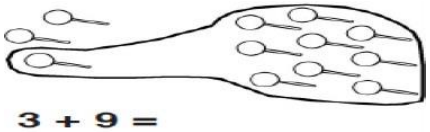


6 + 5 = 11



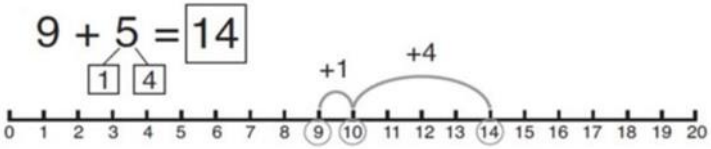
Start with the bigger number and use the smaller number to make 10.

</td>
<td>



Use pictures or a number line. Regroup or partition the smaller number to make 10.

From Year 2



</td>
<td>

7 + 4= 11

If I am at seven, how many more do I need to make 10. How many more do I add on now?

</td>
</tr>
<tr>
<td>

Adding three single digits

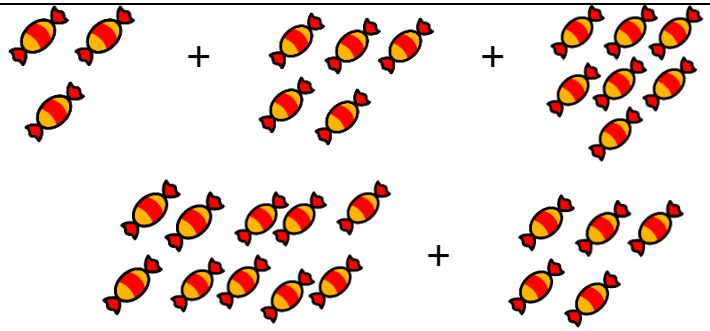
Pupils will use this method from Year 1 onwards

</td>
<td>

4 + 7 + 6= 17
Put 4 and 6 together to make 10. Add on 7.

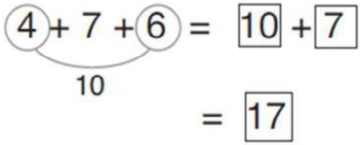
Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit. Also use tens frames.

</td>
<td>



Add together three groups of objects. Draw a picture to recombine the groups to make 10.

</td>
<td>



Combine the two numbers that make 10 and then add on the remainder.

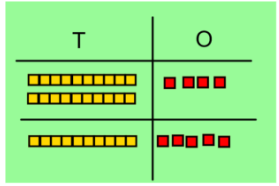
</td>
</tr>
<tr>
<td>

Column method- no regrouping

Pupils will use this method from Year 2 onwards

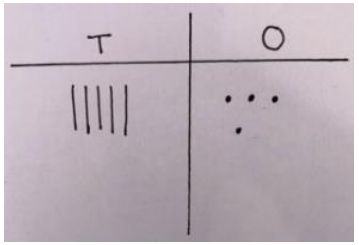
</td>
<td>

24 + 15=
Add together the ones first then add the tens. Use the Base 10 blocks to visualise this. We use the terms ‘chips’ and ‘peas’ to describe the base 10 blocks.



</td>
<td>

After practically using the base 10 blocks, children can draw the ‘chips and peas’ to help them to solve additions.



</td>
<td>

Calculations

21 + 42 =

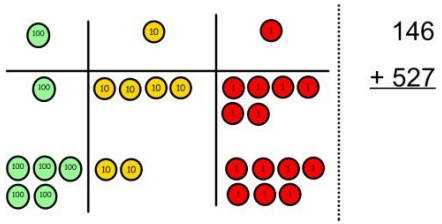
21
+ 42

</td>
</tr>
</table>

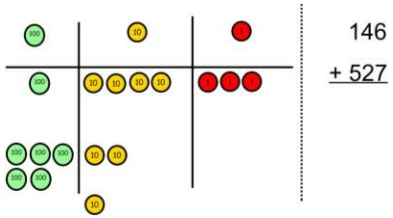
Column method- regrouping

Pupils will use this method from Year 2 onwards

Make both numbers using base 10 (from Year 2) or place value counters (from Year 4).



Add up the units and exchange 10 ones for one 10.

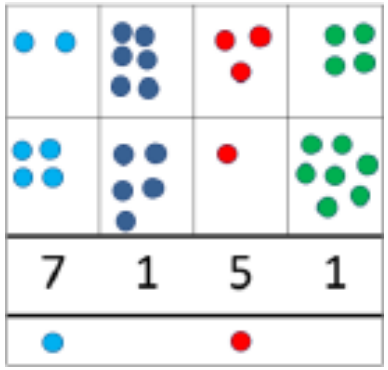


Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictoral representation of the columns to further support their learning and understanding.



Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

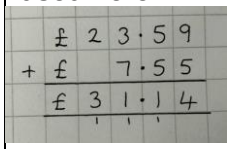
$$\begin{array}{rcrl} 20 & + & 5 & \\ \underline{40} & + & \underline{8} & \\ 60 & + & 13 & = 73 \end{array}$$

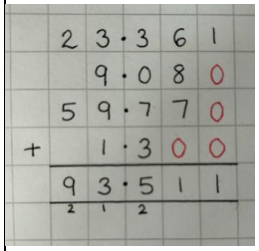
We would encourage the larger number to be placed at the top of a calculation.

$$\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ \hline 1\ 1 \end{array}$$

As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

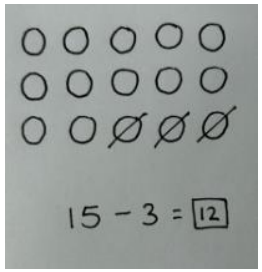
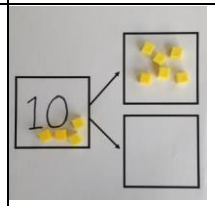
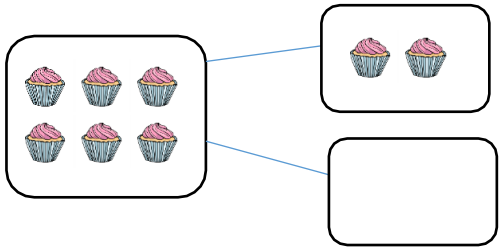
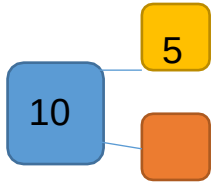
$$\begin{array}{r} 72.8 \\ +\ 54.6 \\ \hline 127.4 \\ \hline 1\ 1 \end{array}$$

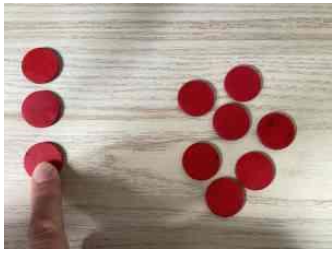
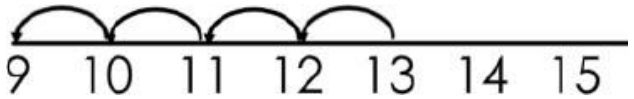
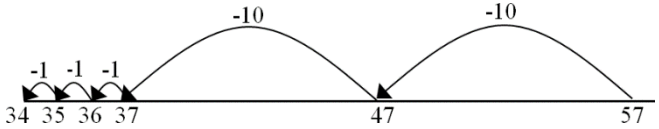
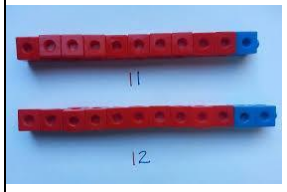
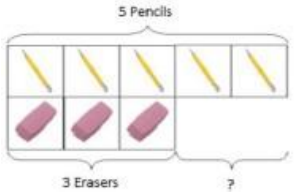
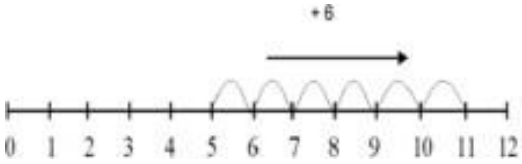
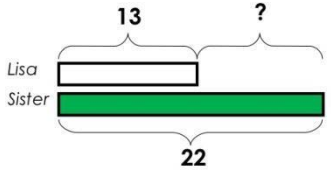


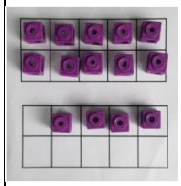
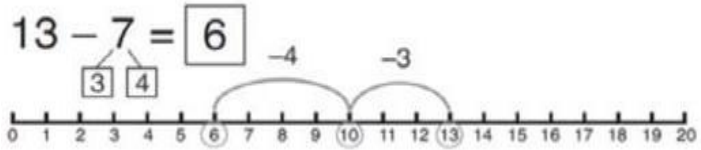
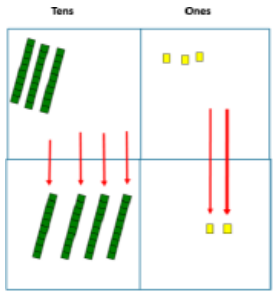
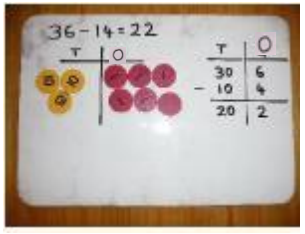
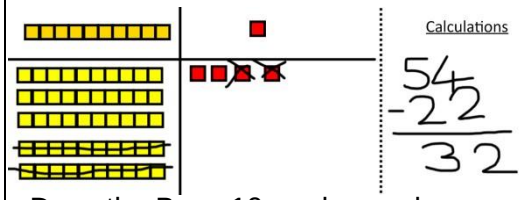
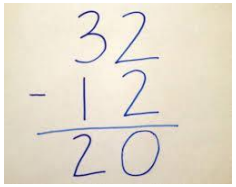


The decimal point should be positioned as shown, on the dividing line between two squares.

Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones *Pupils will use this model from EYFS onwards.*	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4 	Cross out drawn objects to show what has been taken away. 	18 – 3 = 15 8 – 2 = 6
Part Part Whole Model *Pupils will use this model from EYFS onwards.*	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 = 6 | ? 10	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.

Counting back *Pupils will use this model from Year 1 onwards.*	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones, or use a Rekenrek. 13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. Line the counters up to help with subitizing. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line (from Year 2).  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.
Find the difference *Pupils will use this model from Year 1 onwards.*	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on or back to find the difference. **Comparison Bar Models** Draw bars to find the difference between 2 numbers. *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.*  	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.

Make 10 *Pupils will use this model from Year 1 onwards*	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and take away 5 more.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?
Column method without regrouping *Pupils will use this model from Year 2 onwards*	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first.  	 Draw the Base 10 or place value counters alongside the written calculation to help to show working.	47 – 24 = 23 40 + 7 – 20 + 4 20 + 3 As the children gain confidence with understanding the manipulatives, they should begin to record in this manner: This will lead to a clear written column subtraction. 

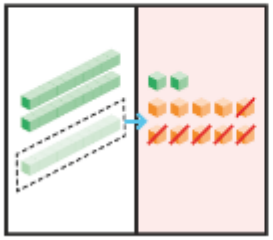
Column method with regrouping

Column method with regrouping

Pupils will use this model from Year 2 onwards

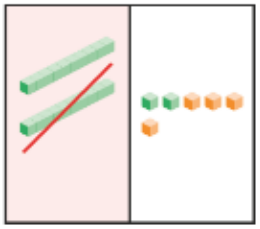
Use Dienes to model regrouping.

Step 1 Regroup 1 ten into 10 ones.
Subtract the ones.
12 ones − 6 ones = 6 ones



	tens	ones
	2 ~~3~~	12 ~~2~~
−	1	6
		6

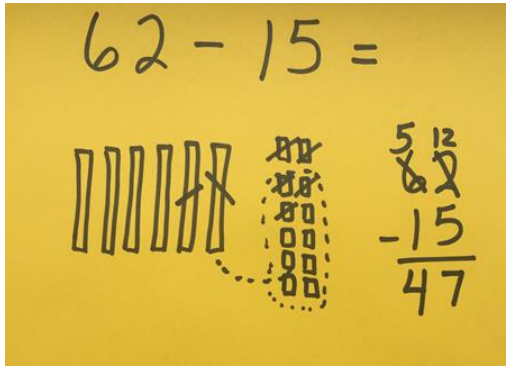
Step 2 Subtract the tens.
2 tens − 1 ten = 1 ten



	tens	ones
	2 ~~3~~	12 ~~2~~
−	1	6
	1	6

32 − 16 = 16

Children can draw a representation of the larger number using place value counters. They can show regrouping by crossing through a ten and drawing an arrow to the 10 ones that have been 'exchanged'. On a whiteboard, they would rub out the ten and replace it with ones.

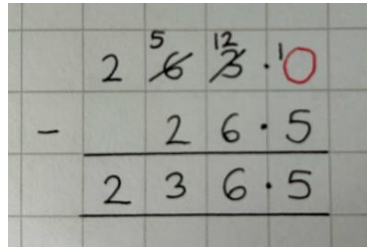


.

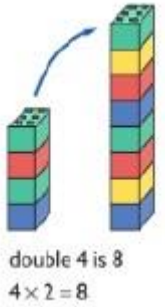
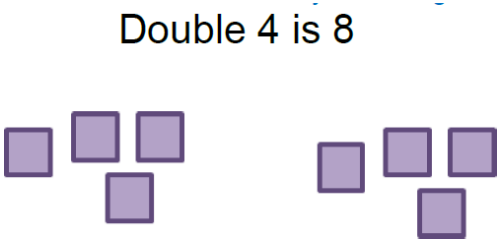
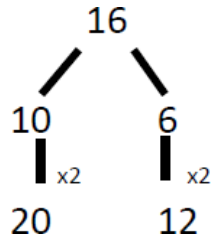
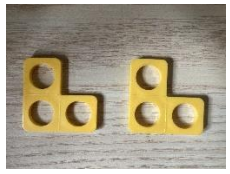
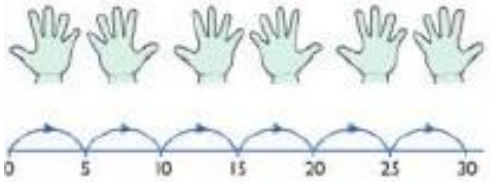
Moving forward the children use a more compact method.

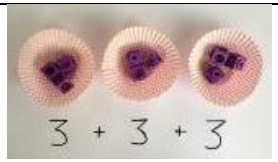
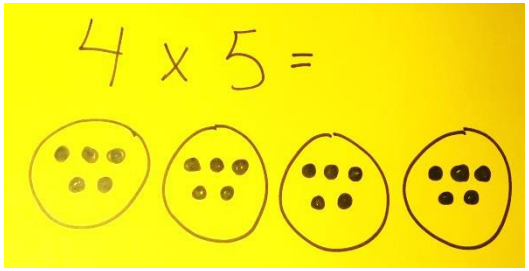
	h	t	o
	~~4~~ 5	1 ~~11~~ 2	~~10~~ 0
−	2	6	9
	2	5	1

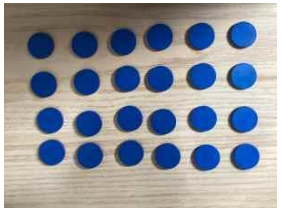
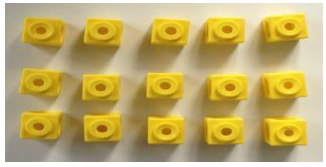
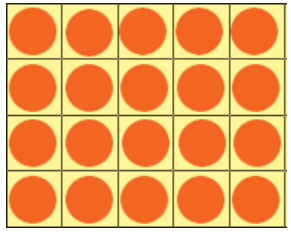
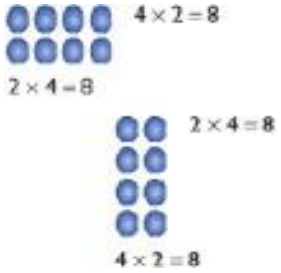
This will lead to an understanding of subtracting any number including decimals.



Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling *Children will use this model from EYFS onwards*	Use practical activities to show how to double a number.  	Draw pictures to show how to double a number.  	 Partition a number and then double each part before recombining it back together.
Counting in multiples *Children will use this model from Year 1 onwards.*	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Repeated addition *Children will use this model from Year 1 onwards.*	  Use different objects to add equal groups.		Write addition sentences to describe objects and pictures. 
Meaning of each factor (When first developing an understanding of multiplication) *Children will use this model from Year 1 onwards.*	When first introducing multiplication, introduce by explaining that first factor tells us how many groups and the second tells us how many in the group. The product is how many there are altogether.  3 groups of 5 flowers = 15 flowers 3 lots of 5 flowers = 15 flowers 3 x 5 = 5 + 5 + 5 = 15	Children can draw pictures to represent the meaning of multiplication sentences: 	After seeing many concrete and pictorial representations, children can move on to saying the meaning of each number in multiplication sentence: 4 x 5 = 20 'There are four groups with 5 in each group which equals 20 altogether'.

Note: Once children have developed a basic understanding of multiplication including its commutative nature (see below), it is not necessary to specify the meaning of each factor. As is the practice in Shanghai and Singapore, either factor can be the multiplier or multiplicand eg. 24 x 3 can mean 24 lots of 3 or the number 24 three times. The language of 'multiplied by' needs to be introduced in Year 2 alongside commutivity.			
Arrays- showing commutative multiplication *Children will use this model from Year 2 onwards.*	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences (mainly developed in Year 3):.  	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

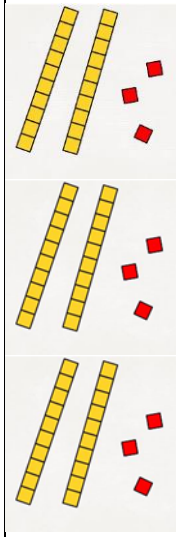
2 digit Multiplication

Children will use this model from Year 3 onwards.

When first introducing multiplying 2 digit numbers, Base 10 'chips and peas' are used to help the children 'see' the whole number that is being multiplied:

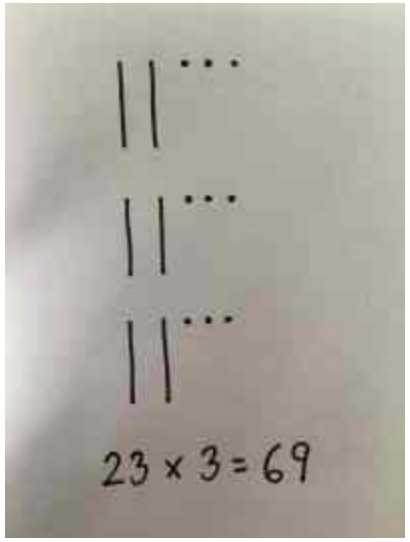
3 x 23 =

means 3 lots of 23:

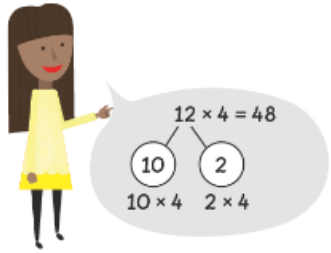


The ones are multiplied first so that if there are more than 9 ones they can be exchanged for a ten.

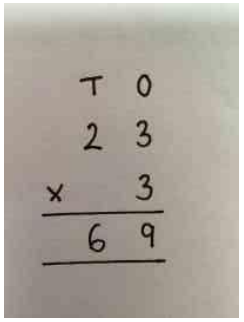
After that a drawn version is used alongside a written method.



The children can use a part-whole diagram to partition the 2 digit number and multiply each part:



Once this is secure, they record the calculations in compacted columns:

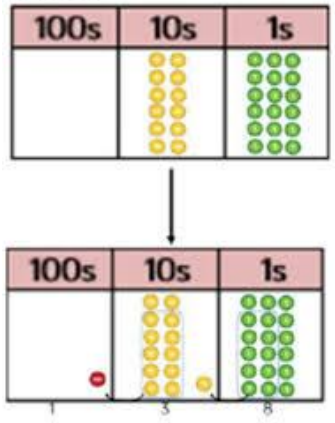


2 and 3 digit multiplication with regrouping

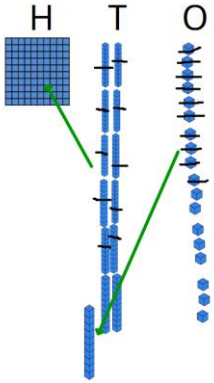
Children will use this model from Year 3 onwards.

Children will use place value counters to represent the numbers:

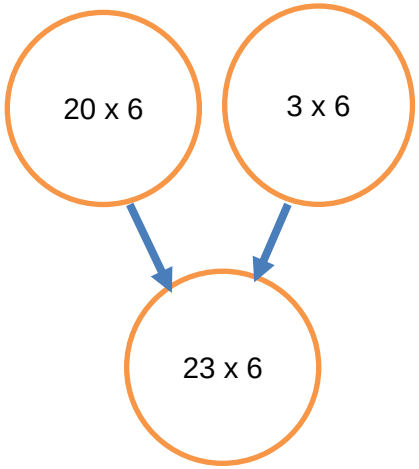
6 x 23



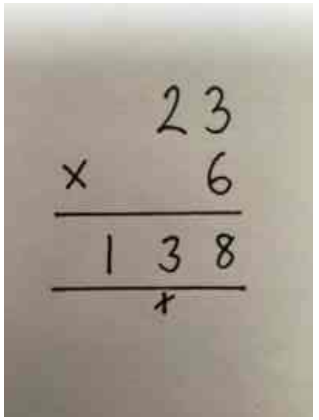
They will also use base 10 equipment to represent the numbers:

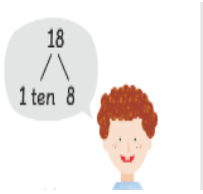


A part whole model can also be used to help the children understand what is happening in this calculation:



A written compacted form is developed:



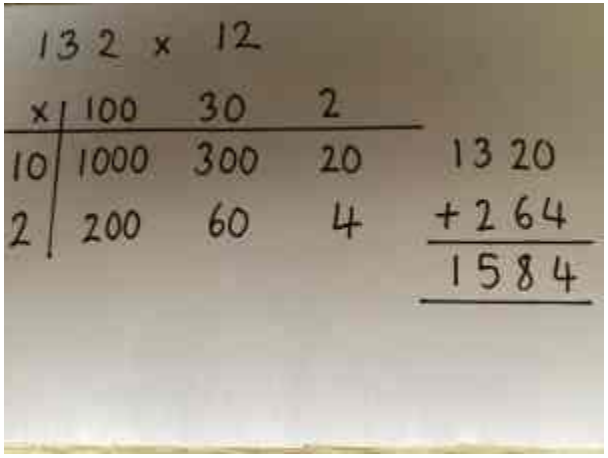


In each Year of KS2 children will should revisit multiplication using the concrete, pictorial and abstract approaches so that they understand the compact, written method.

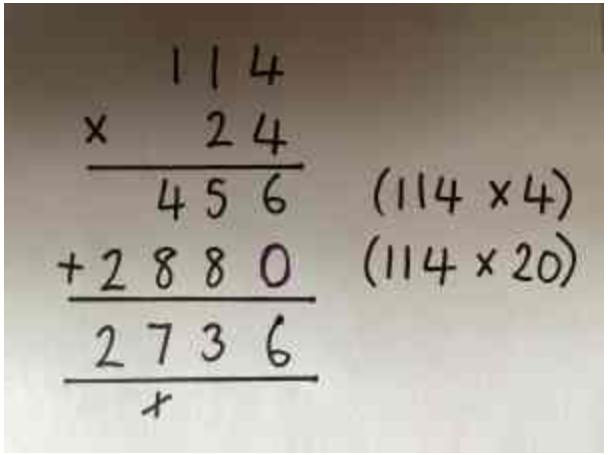
Multiplication: 3 digit by 2 digit

Children will use this model from Year 5 onwards.

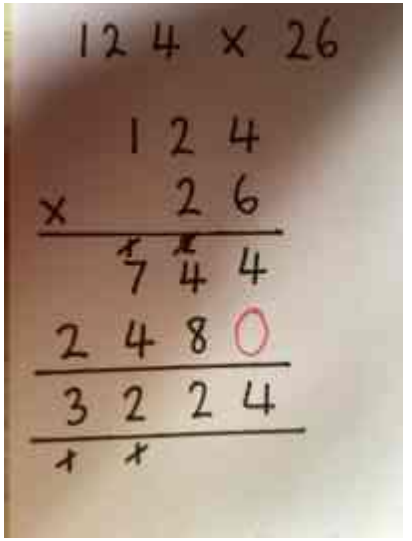
Pupils will use a simple grid filled with place value counters or their own drawings to represent hundreds, tens and ones. With any visual representation in a grid the 3 digit number will always be used first, across the top of the visual representation.



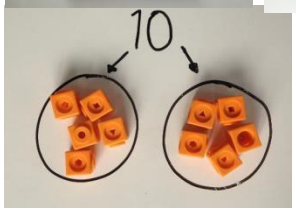
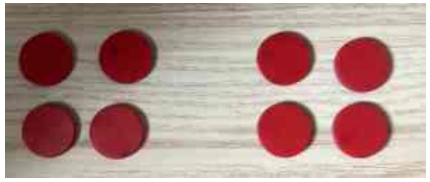
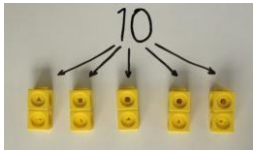
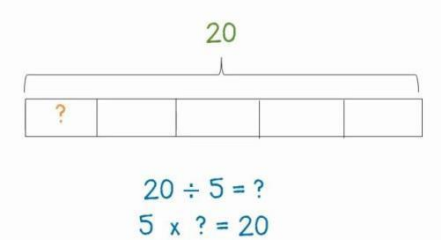
Each part of the calculation is labelled at the side as a reminder of the calculation in that row. Multiply the ones first then the tens. The place value holder 0 is written in a different colour.



When the children understand the calculation it can be compacted further to look like this. The 0 place holder continues to be shown in a different colour as a reminder.



Division

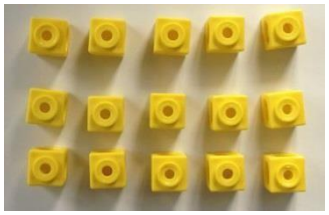
Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects fairly *Children will do this with equipment in EYFS and Year 1 but will begin to use the notation starting in Year 2.*	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities equally.  8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Division as grouping *Children will do this with equipment in Year 1 but will begin to use the notation starting in Year 2.*	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. 'I have 10 biscuits, I give 2 to each child, how many children can get biscuits?  I have 12 chairs. I put 4 chairs around each table, how many tables do I need? 	Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  Use a stem sentence: 20 divided by 5 is 4.	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

<table>
<tr>
<td>

Division within arrays

Children will use this model from Year 2 onwards.

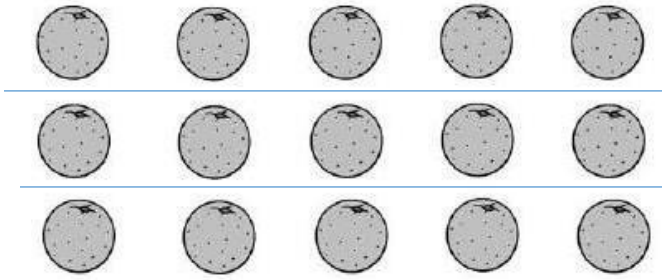
</td>
<td>



Link division to multiplication by creating an array and thinking about the number sentences that can be created.

Eg 15 ÷ 3 = 5 5 x 3 = 15
15 ÷ 5 = 3 3 x 5 = 15

</td>
<td>



Draw an array and use lines to split the array into groups to make multiplication and division sentences.

</td>
<td>

Find the inverse of multiplication and division sentences by creating four linking number sentences.

7 x 4 = 28
4 x 7 = 28
28 ÷ 7 = 4
28 ÷ 4 = 7

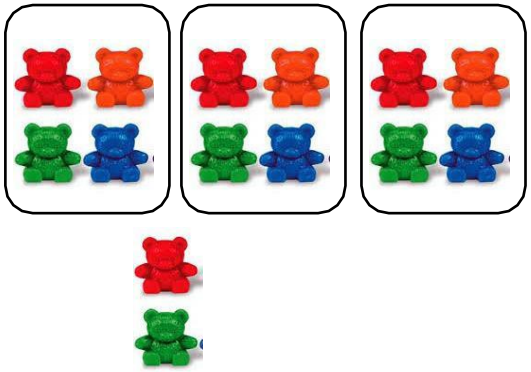
</td>
</tr>
<tr>
<td>

Division with a remainder

(Years 3-6)
** children working at greater depth in Year 2 will cover this.*

</td>
<td>

14 ÷ 3 =
Divide objects between groups and see how much is left over



</td>
<td>

Draw dots and group them to divide an amount and clearly show a remainder.

</td>
<td>

Complete written divisions and show the remainder using r.

29 ÷ 8 = 3 r 5

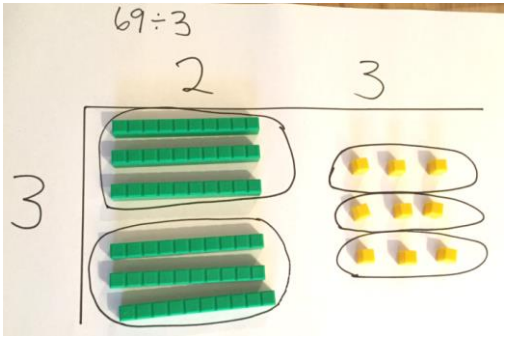
</td>
</tr>
</table>

Short division

Children will use this model from Year 3 onwards

When children do not know their times tables well, a 'sharing' method works well.

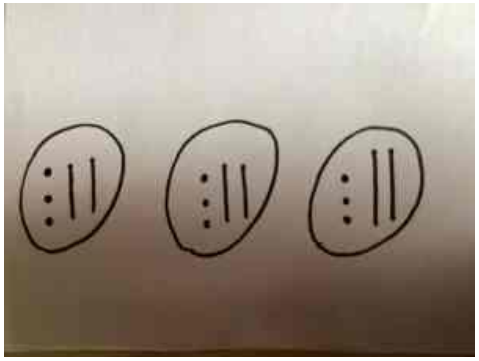
When dividing 2 digit numbers children begin by representing the number with Dienes. They then see how many groups of the divisor they can make:



I can make 2 groups of 3 tens. I can make 3 groups of 3 ones.

Pupils should start each time with the largest place value ie the tens.

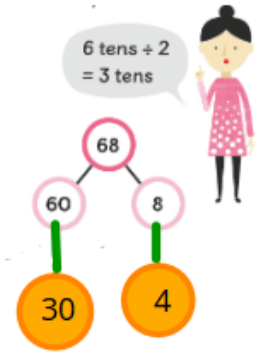
Children can use drawings to represent the Dienes (or they can draw place value counters) and see how they can be grouped:

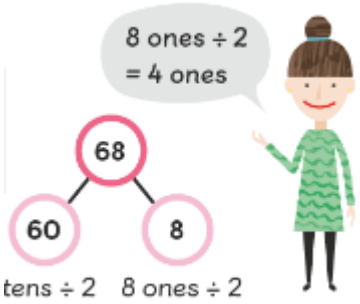


Encourage them to move towards counting in multiples to divide more efficiently.

This can link to a mental method of partitioning:

68 ÷2 =

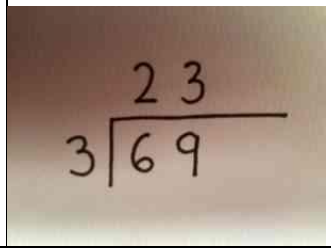




Children will use this model from Year 4 onwards

Once pupils know their times tables grouping using the 'bus stop' method works well.

The compact method should continue to be taught alongside using Dienes or place value counters until understanding is secure. We call this the 'bus stop' method.



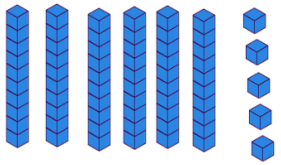
Short division with regrouping

Children will use this model from Year 3 onwards.

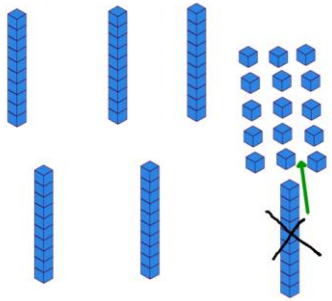
In Year 3 pupils explore calculations like 65 ÷ 5 by making groups of the divisor and discovering that the 'extra' 10 can be regrouped with the ones. How many groups of 5 in 6 (tens)?

65 ÷ 5 =

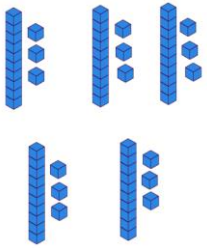
Step 1: Display the amount using Base 10.



Step 2: Share the tens first (into 5 groups in this case). Exchange any left over tens into ones.



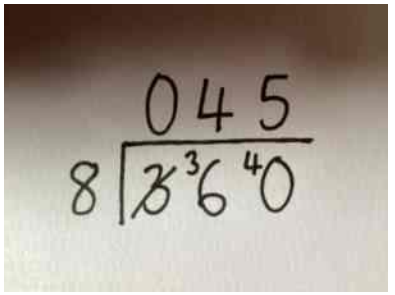
Step 3: Share the ones.



In Years 4 to 6 pupils can make drawings of the Dienes ('fish, chips and peas') or place value counters and group them to model short division. The 'bus stop' algorithm should be introduced alongside the drawing.

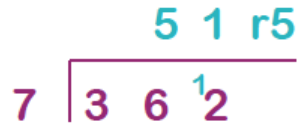
54 ÷ 4 = 13 r 2

Begin with divisions that divide equally with no remainder.



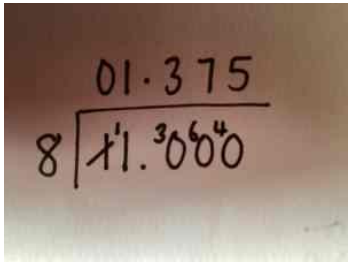
Move onto divisions with a remainder.

362 ÷ 7 =



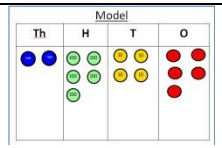
362 ÷ 7 = 51 r5

Finally move into decimal places to divide the total accurately.



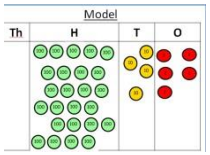
Long division

Children will use this calculation from Year 6 onwards. Division with decimals is taught at the very end of Year 6.



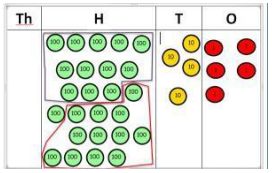
2544 ÷ 12
How many groups of 12 thousands do we have?
None

Exchange 2 thousand for 20 hundreds.



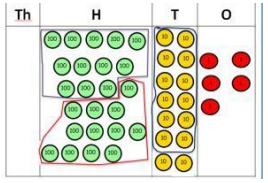
$$12\overline{)2544}\quad 0$$

How many groups of 12 are in 25 hundreds? 2 groups. Circle them.
We have grouped 24 hundreds so can take them off and we are left with one.



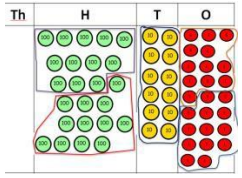
$$12\overline{)2544}\quad 02$$
$$24$$
$$1$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2



$$12\overline{)2544}\quad 021$$
$$24$$
$$14$$
$$12$$
$$2$$

Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2



$$12\overline{)2544}\quad 0212$$
$$24$$
$$14$$
$$12$$
$$24$$
$$24$$
$$0$$

Instead of using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books.

Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process.

Children should be encouraged to think of long division as a way of keeping track of the calculations they are already doing mentally when they use the short division method.
Long division should be used when the divisor is a 2 digit number where the mental calculations become too complex to keep track of.

432 ÷ 15 becomes

$$15\overline{)432.0}\quad 28.8$$
$$30$$
$$132$$
$$120$$
$$120$$
$$120$$
$$0$$

They can write a list of multiples of the divisor to help them or use estimation to see how many groups of 15 there are at each step.